ANALYSIS OF THE FILM CONDENSATION PROCESS OF VAPOR ON A VERTICAL FINELY SERRATED SURFACE

N. V. Zozulya and V. A. Karkhu

The process of film condensation of vapor on vertical finely serrated surfaces is investigated. The analysis shows that owing to the specific conditions under which the distribution of the condensed film along the cooling surface is in the main affected by surface tension, the effectiveness of such surfaces may be several times higher than that of plain tubes.

It was noted in [1, 2] that owing to the presence of surface forces a fine wavy surface of vapor condensation results in a distribution of the liquid film such that the coefficient of heat transfer is considerably improved over that of a plane tube.

This paper is devoted to an analysis of the laws of vapor condensation along vertical surfaces with fine longitudinal serrations of trapezoidal form. Such finely serrated surfaces (Fig. 1b) have certain advantages over fine wavy surfaces consisting of two tangent semicircles (Fig. 1a) because of wide possibilities for altering the shape of their elements, and owing to their comparatively simple manufacturing technology.

Let us consider the phenomena occurring under conditions of maximum effect of surface tension whose magnitude depends primarily on the curvature of the heat-exchange surface. If a Weber number $W = \sigma/R_1^2\rho \geq 10$ is assumed, the physical process may be represented by the following model:

1) If the surface forces are of one order of magnitude greater than gravity ($W \geq 10$), it can be assumed that the liquid condensed on the surface of a serration flows by the shortest path into the recess under the influence of the surface tension only;

2) Because of this flow a layer of condensate several times thicker than at the top of a serration tooth collects at the bottom of a recess. It thus becomes possible to disregard the condensation of vapor along a recess, and to consider only the hydrodynamic problem of laminar flow of liquid in the latter.

The general problem of heat transfer and of the hydrodynamics of condensate-film flow along elements of the serration surface may, on these assumptions, be reduced to two separate problems: first, the determination of thickness of the condensate at the serration top and of the velocity of its flow into the recess, and, second, the determination of both the flow velocity in the recess under the action of gravity and the zone of total flooding of the recess.

1. Condensation of Vapor at the Tip of a Serration. Our analysis of the film condensation process of vapor is based on Nusselt's premises [3] in the theory of film condensation of steam on a vertical surface:

1) The condensate forms a continuous film on the wall, and the heat flux intensity is determined by the thermal resistance of that film,

$$q = \frac{\lambda}{\delta_x}(t_s - t_w) \tag{1.1}$$

2) The temperature throughout the trapezoid height is assumed constant. This condition can be satisfied only within certain limits, since a decrease of the ratio of serration base to height may, owing to the effect of finite thermal conductivity, result in a marked temperature gradient along the height of a serration.

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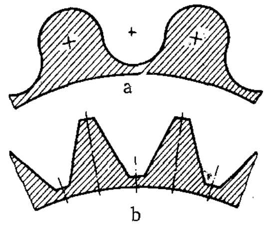

Fig. 1. Variations of finely serrated surfaces.

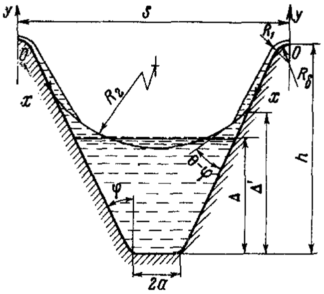

Fig. 2. An element of serration surface under conditions of vapor condensation.

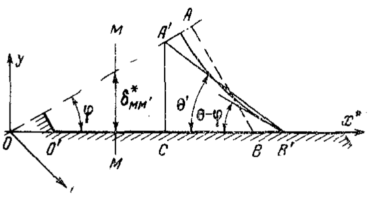

Fig. 3. Calculation of the velocity of liquid motion in the recess.

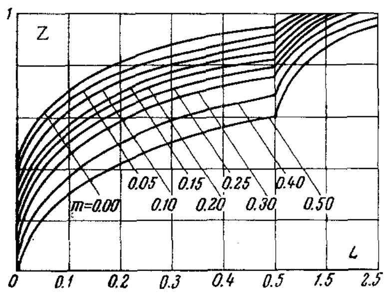

Fig. 4. Curves of the level of channel flooding Z in terms of the dimensionless coordinate L for various parameters m.

3) Temperatures at the film boundaries are equal to the temperatures, respectively, of the wall t_w and of the saturated vapor t_s at its free surface, with the heat transfer coefficient defined by the inequality $\alpha = \lambda/\delta_x$.

4) The vapor is considered to be stationary.

The presence of a film of condensate on a wetted wall surface generally depends on the combined action of forces of gravity, friction, inertia, and surface tension at the liquid boundary.

The differential equation of motion applicable to the plane problem of flow of liquid into the recess (Fig. 2) considered here is written as

$$w_x \frac{\partial w_x}{\partial x} + w_y \frac{\partial w_y}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\rho - \rho''}{\rho} g \cos(g, x) + \nu\left(\frac{\partial^2 w_x}{\partial x^2} + \frac{\partial^2 w_y}{\partial y^2}\right) \quad (1.2)$$

In light of the above statements let us consider the laminar flow of condensate along the protruding part of a serration under the action of surface tension and friction only. This leads to a considerable simplification of Eq. (1.2) which becomes

$$-\frac{1}{\mu}\frac{\partial p}{\partial x} + \frac{\partial^2 w_y}{\partial y^2} = 0 \quad (1.3)$$

After integration we obtain

$$w_y = \frac{1}{2\mu}\frac{\partial p}{\partial x} y^2 + C_1 y + C_2$$

From the boundary conditions for the flow of condensate in the film we have

$$y = 0, \qquad w_y = 0, \qquad C_2 = 0$$

$$y = \delta(x), \qquad \frac{\partial w_y}{\partial y} = 0, \qquad C_1 = -\frac{1}{\mu}\frac{\partial p}{\partial x}\delta_x$$

For the velocity field in a cross section of the film at distance x from the serration tip we obtain the following equation:

$$w_y = \frac{1}{\mu}\frac{\partial p}{\partial x}\left(\frac{y^2}{2} - \delta_x y\right) \quad (1.4)$$

The condensate rate of flow over the serration tip, averaged over the film cross section, is

$$\langle w_y \rangle = \frac{1}{\delta_x}\int_0^{\delta_x} w_y\, dy = \frac{\delta_x^2}{3\mu}\left|\frac{\partial p}{\partial x}\right| \quad (1.5)$$

The condensate rate of flow across a section at x is

$$G_x = \rho \langle w_y \rangle \delta_x = \frac{\rho\delta_x^3}{3\mu}\left|\frac{\partial p}{\partial x}\right|$$

The rate of condensate flow over a serration through a section at distance dx downstream of the flow is increased by

$$dG_x = \frac{\rho}{\mu}\left|\frac{\partial p}{\partial x}\right| \delta_x^2 d\delta_x \quad (1.6)$$

This increase is due to condensation of vapor. Substituting $dQ = rdG_x$ into Eq. (1.1), we obtain

$$dG_x = \frac{\lambda}{r\delta_x}(t_s - t_w)\,dx \tag{1.7}$$

Equating the right-hand sides of (1.6) and (1.7), we derive the equation

$$\frac{\rho}{\mu}\left|\frac{\partial p}{\partial x}\right|\delta_x^3\,d\delta_x = \frac{\lambda}{r}(t_s - t_w)\,dx$$

which after integration yields

$$\delta_x = \left(\frac{4\mu\lambda(t_s - t_w)x}{\rho r\,|\partial p/\partial x|}\right)^{1/4} \tag{1.8}$$

which is the local thickness of film on the lateral surface of the trapezoid at distance x from the serration tip. Since the pressure gradient $\partial p/\partial x$ is negative (the recess is a region of lower pressure), its absolute value is used in Eq. (1.8).

If we define $(\partial p/\partial x)$ as the ratio of pressure drop from tip to recess

$$\Delta p = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

to the total path of liquid flow along a serration, which depends on the extent of local flooding of the recess (Fig. 2) and is

$$\Delta x = \frac{h - \Delta}{\cos\varphi}$$

we find that

$$\left|\frac{\partial p}{\partial x}\right| \approx \frac{\Delta p}{\Delta x} \approx \frac{\sigma\cos\varphi}{R_\delta(h - \Delta)} \tag{1.9}$$

It is assumed here that at the tip $R_1 \approx R_\delta$ and in the recess $R_2 \to \infty$.

Substituting the derived value of $\partial p/\partial x$ into Eq. (1.8), we obtain

$$\delta_x = \left(\frac{4\mu\lambda(t_s - t_w)R_\delta(h - \Delta)x}{\rho r\sigma\cos\varphi}\right)^{1/4} \tag{1.10}$$

2. Motion of Condensate in the Recess. Let us determine the interdependence between the rate of condensate flow over the serration into the recess and the velocity of fluid motion along the latter, as it is being gradually filled. We confine our consideration to the case of laminar flow in the recess, and assume the absence of vapor condensation in it. It should be, first of all, noted that contact angles θ of surface wetting by the liquid have an appreciable effect on the shape of the free surface of the liquid flowing in the recess, which may be dimensionally close to a capillary, and consequently on the magnitude of the wetted perimeter and the rate of flow. Owing to the wetting effect a concave meniscus is formed in the recess filled by a layer of liquid to a height of Δ (Fig. 2), and this makes the liquid rise by a certain additional height $\Delta' - \Delta$ depending on the wetting angle.

From Fig. 2 it can be found by conventional geometry that

$$\Delta' = \xi\Delta, \quad \xi = \left(1 + 1.3\frac{1 - \sin\theta}{\cos\theta}\operatorname{tg}\varphi\right)^{1/2} \tag{2.1}$$

In Fig. 3 is shown a half-section of the recess lying between the axis of symmetry and the trapezoid flank filled with liquid to a certain level Δ' with the effect of surface wetting taken into consideration. For simplicity, the curve of the film upper surface is replaced by a straight line A'B' at a certain angle $\theta' \approx \theta - \varphi + \beta$ to the flank with β on the order of 10-20° depending on the conditions of wetting.

Let us visualize the motion of the fluid film in the vertical direction Ol in this half-channel as a certain family of plane-parallel flows along the flank of the trapezoid with the width O'B' of the stream varying with the coordinate l of flooding and the film thickness δ^* varying over the width of this stream. It is assumed that, for example, at a certain point x^* (the coordinate l of flooding is fixed) there is in section MM' a plane flow with film thickness equal δ^*.

We restrict our consideration of this model of flow to the case in which the dimension a (half-width of the recess bottom) is small and, consequently, its effect on the liquid flow pattern in the channel may be neglected.

For a plane laminar flow of liquid along a wetting surface Nusselt had established the well-known parabolic law for the velocity distribution across the film thickness. We assume that in every cross section (assumed normal to the channel side wall) prevails its own law of semiparabolic distribution of velocity u across the thickness of film (from zero at the wall to a certain limit value at the axis of symmetry of the channel depending on local δ^*), which in our notation is of the form

$$u(y, x^*) = \frac{\rho \cos(g, l)}{\mu} \left[\delta^*(x^*) y - \frac{y^2}{2} \right] \tag{2.2}$$

The mean flow rate of liquid across the recess is

$$\langle u(y, x^*) \rangle = \frac{1}{F} \iint u(y, x^*)\, dy\, dx^* \qquad \left(F = \iint dy\, dx^* \right) \tag{2.3}$$

Here F is the cross-sectional area of liquid in the channel.

From this, using the notation

$$z = \frac{\Delta}{h}, \quad n = \frac{\operatorname{tg} \varphi}{\operatorname{tg}(\Theta - \varphi + \beta)}, \quad m = \frac{a}{\xi h \operatorname{tg} \varphi} \tag{2.4}$$

and integrating (for $m \leq 0.5$), we obtain

$$\langle u(y, x^*) \rangle = \frac{\xi^2 \rho \cos(g, l) \operatorname{tg}^2 \varphi h^2 (z + m)^2}{6 \cos^2 \varphi (1 + n) \mu} \tag{2.5}$$

The value of $m \leq 0.5$ was chosen to accord with the previously introduced limitation of the dimension a of the trapezoid. The flow rate of liquid across half of the recess is

$$G_l = \rho u F = \frac{\xi^4 \cos(g, l) \operatorname{tg}^3 \varphi \rho^2 h^4 (z + m)^4}{12 \cos^4 \varphi (1 + n)^3 \mu} \tag{2.6}$$

The variation of the flow rate is

$$dG_l = \frac{\xi^4 \cos(g, l) \operatorname{tg}^3 \varphi \rho^2 h^4 (z + m)^3}{3 \cos^4 \varphi (1 + n)^3 \mu} dz \tag{2.7}$$

The change of flow rate along a section dl of the flooded path is, on the other hand, due to the inflow of condensate from the surface of a serration at a rate

$$dG_l = \langle w_y^\circ \rangle \rho \delta_x^\circ dl \tag{2.8}$$

Here $\langle w_y^\circ \rangle$ and δ_x° are, respectively, the mean velocity (1.5) and the thickness of the liquid film (1.10) running off a serration at the boundary of the flooding layer.

Introducing the concept of dimensionless coordinate of path L normalized with respect to H (m),

$$L = \frac{l}{H}, \qquad H = \frac{0.35 \xi^4 \cos(g, l) \operatorname{tg}^3 \varphi \rho^{7/4} h^{7/2} R_\delta^{1/4} r^{3/4}}{(1 + n)^3 \cos^{7/2} \varphi \sigma^{1/4} \mu^{3/4} \lambda^{3/4} (t_s t_w)^{3/4}} \tag{2.9}$$

we derive for the flow of liquid in the recess the following differential equation:

$$\frac{(z + m)^3}{\sqrt{1 - z}} dz = dL, \quad m \leqslant 0.5 \tag{2.10}$$

which is readily integrable

$$C_1\,(1-z)^{1/2} - C_2\,(1-z)^{3/2} + C_3\,(1-z)^{5/2} - C_4\,(1-z)^{7/2} + B = L, \quad m \leqslant 0.5 \tag{2.11}$$

$$C_1 = 2 + 6m + 6m^2 + 2m^3$$
$$C_2 = {}^2/_3\,(3 + 6m + 3\,m^2)$$
$$C_3 = {}^2/_5\,(3 + 3m)$$
$$C_4 = {}^2/_7$$

The constant of integration is determined by the boundary condition (for $l = 0$) as

$$B = -C_1 + C_2 - C_3 + C_4 \tag{2.12}$$

Equation (2.11) makes it possible to determine the distribution pattern of the thickness of the condensate film in a flow along the vertical surface of a recess of trapezoidal form, and to find the coordinate l of the critical point at which the groove becomes completely flooded by the liquid and beyond which the present analytical investigation no longer applies.

A number of curves $Z = f(L)$ corresponding to Eq. (2.11) is shown in Fig. 4 in dimensionless coordinates. These represent variation of the relative film thickness z in terms of coordinate L depending on the "form resistance" m which depends on the relation between the basic geometric dimensions of the profile selected for the vapor condensing surface.

3. The Coefficient of Heat Transfer. The determination of the mean coefficient of heat transfer over a certain length $L_0 \leq L_*$ (L_* is the coordinate of the point at which the recess is completely flooded) does not, obviously, necessitate a calculation of heat removal over the whole perimeter of the tube, since it is sufficient to investigate a single elementary section of a width equal to the serration pitch S.

For given conditions of the process and geometry of the surface the coefficients H and m are first determined from (2.9) and (2.4), respectively, and from Fig. 4 or, alternatively, the level z of recess flooding at $L = L_0$ is determined from Eq. (2.11). Having found the level of flooding z, the flow rate of liquid through the end section of the recess

$$G = \frac{0{,}167\xi^4 \cos(g,\ l)\, \mathrm{tg}^3\varphi\rho^2 h^4 (z + m)^4}{(1 + n)^3 \cos^4\varphi\mu} \tag{3.1}$$

is found with the use of formula (2.6), which gives the flow rate through half of the recess.

The heat of vapor condensation Q = Gr is transmitted to the wall through the liquid film in accordance with the averaging [formula] of the form

$$Q = \alpha^* (t_s - t_w)\, F_s \tag{3.2}$$

Here F_s is the total surface area of heat exchange of an element of width s.

From this the sought heat transfer coefficient is determined as

$$\alpha^* = \frac{Gr}{(t_s - t_w)\, F_s} \tag{3.3}$$

The above analytical investigation of the laws governing film condensation of vapor on finely serrated surfaces permits us to assess the effectiveness of such surfaces in the domain restricted by the requirements ($W \geq 10$; $m \leq 0.5$) specific to the selected physical model of this phenomenon.

Preliminary calculations of heat transfer through certain easily produced variants of serrated surfaces show that the heat transfer coefficient for these is 1.5-2 times higher than for plane surfaces, while the total heat removal, owing to the increased surface area by the addition of serrations, exceeds that of plane tubes by a factor of 4-6.

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